

A Fast Optical Propagation Technique for Modeling Micro-Optical Systems

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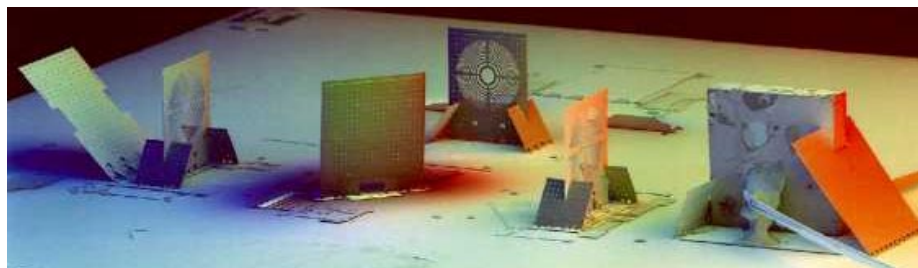
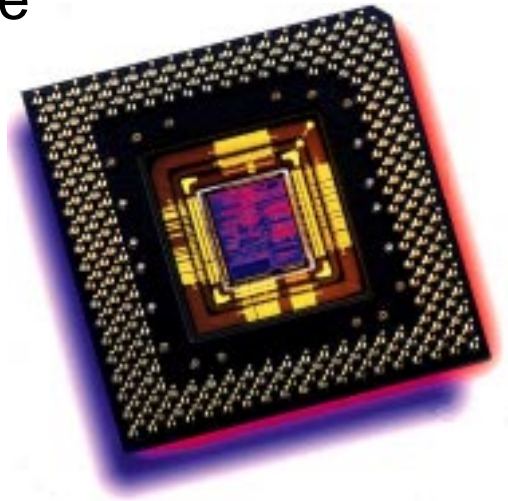


Overview

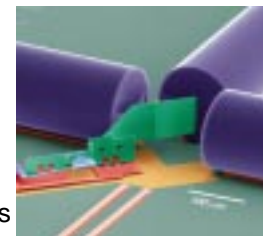
- Multi-domain micro-systems
- System-level CAD for mixed-signal, multi-domain systems
- Optical propagation
- Angular spectrum technique
- GLV simulation and analysis
- Summary and future work

Optical Microsystems

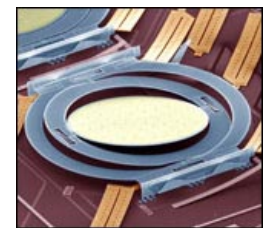
- Applications: switching, projection, scanning, display, printing, sensing, modulating, and data storage
- Optics is good for information transport
 - High parallelism and small skew
 - High speed with low crosstalk
 - Low power and low latency
 - Fan-in/fan-out and permutations
- Electronics is good for computation
- Micro-mechanics is good for interactions with the “real world”
 - Sensing, actuating, switching



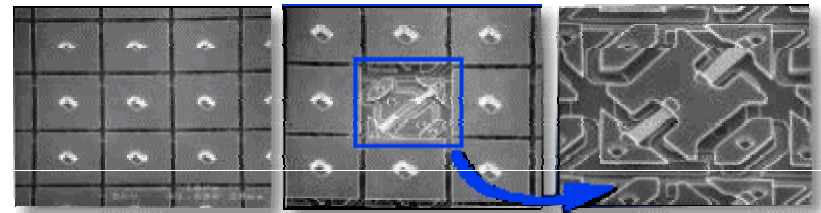
UCLA - Integrated Free-Space Optical Disk Pickup Head



Bell Labs



Lucent



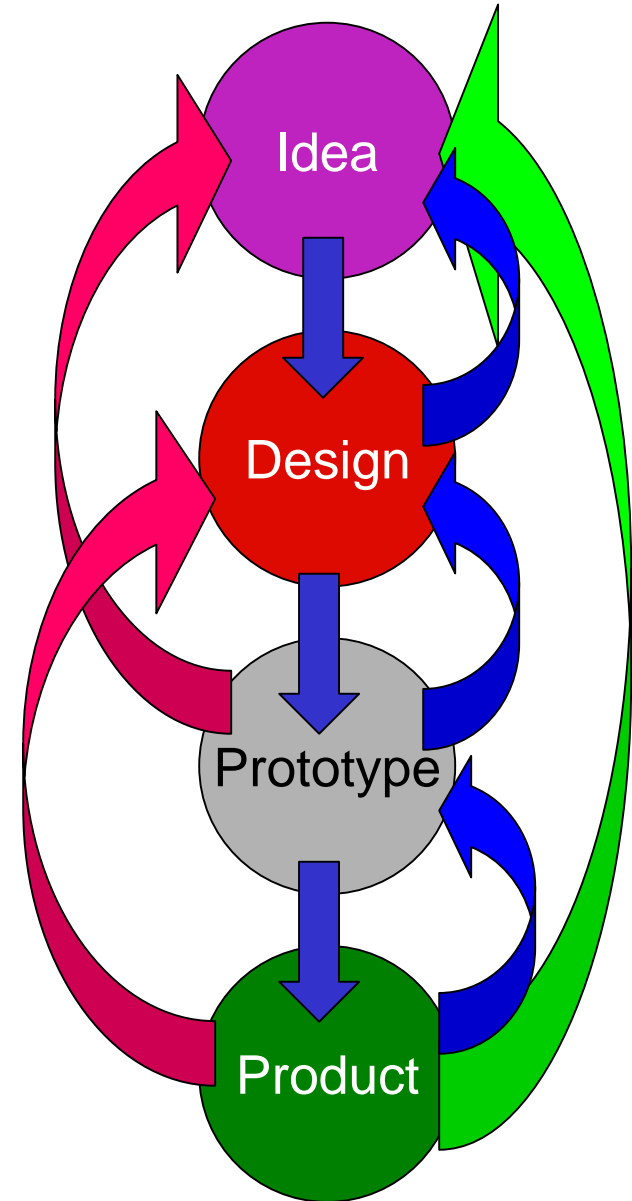
Texas Instruments - DMD

Mixed-Signal, Multi-Domain CAD

- CAD for optical MEM systems
 - Reduce costly prototyping
 - Test feasibility and effectiveness
 - Perform architecture vs. technology trade-offs

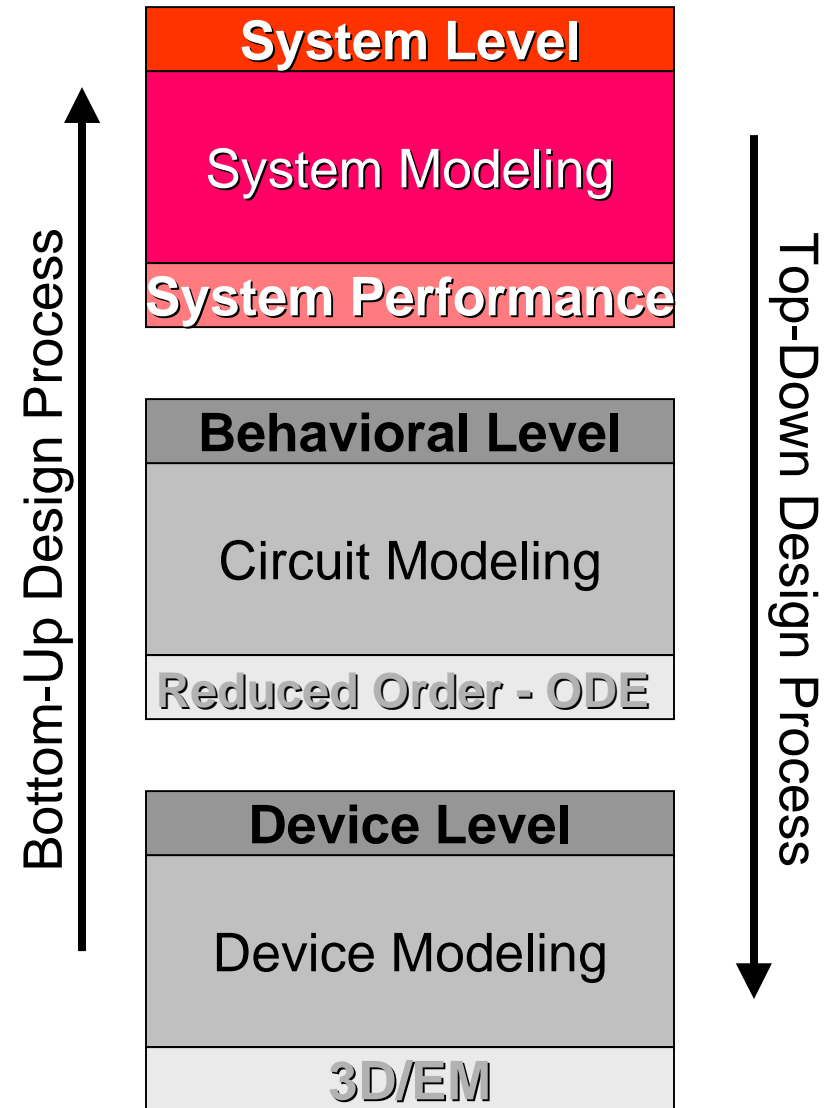
Goal: reduce the time-to-market and find the unexpected design flaw

- Difficulties
 - Mixed domains & interactions
 - *Optical*
 - Electrical
 - Mechanical
 - Modeling levels
 - Device
 - Behavioral
 - *System*



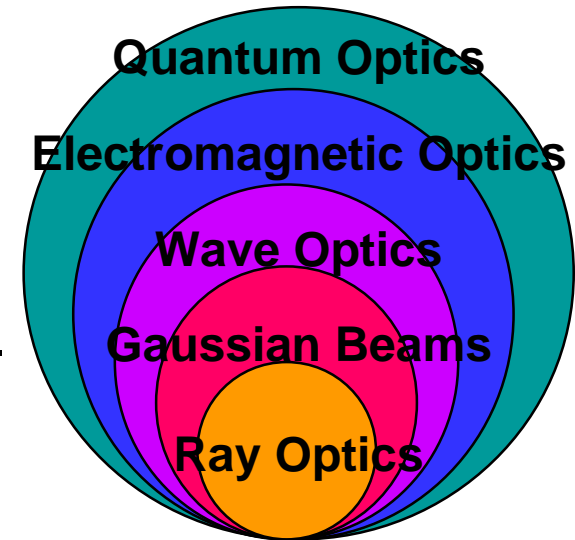
System-Level Simulation

- Ensemble of component behavioral models
- Fast solvers at component/behavioral level
- Domain specific signal propagation models
- Global discrete event dataflow
- Ensemble performance measures:
 - BER
 - optical/electrical crosstalk
 - insertion loss
 - packaging/alignment tolerances
 - thermal effects
 - etc.



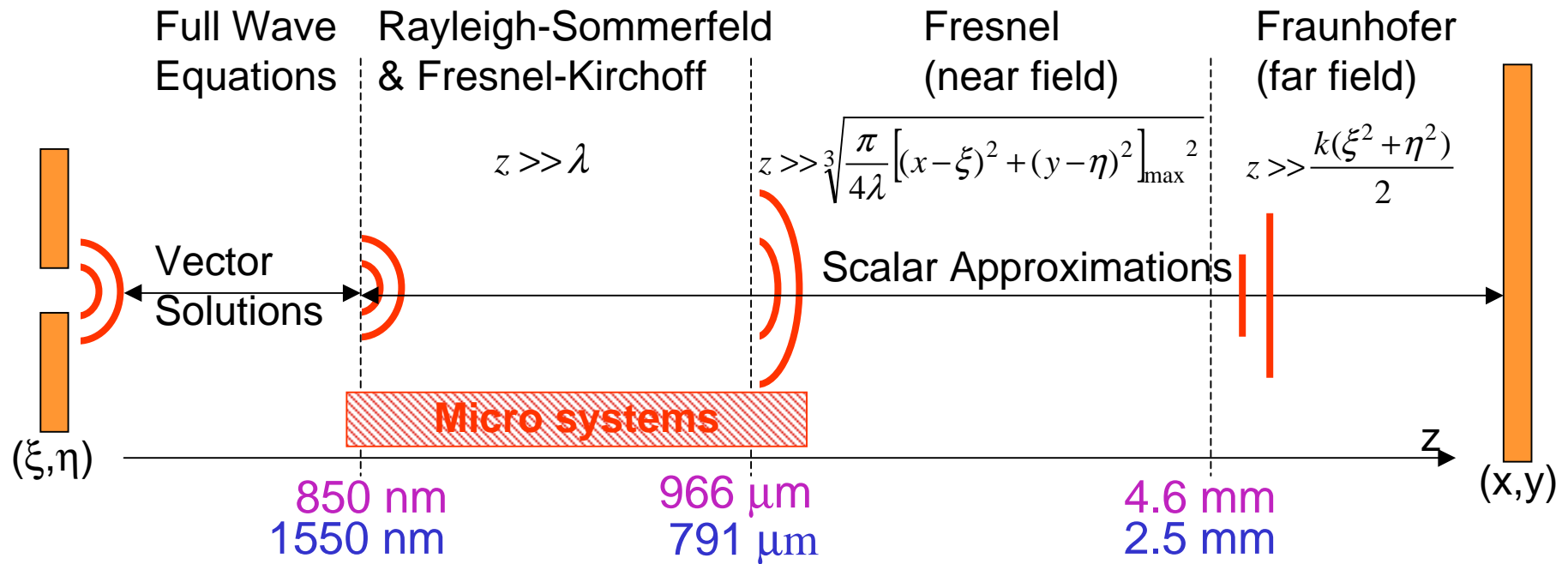
Optical Propagation Models

- Geometric Ray Propagation
 - Positions and angles
 - No optical signal characterization
- Gaussian Propagation
 - 8 scalar parameters - z_0 , x , y , λ , etc.
 - Fast computation - no explicit integration at each component
 - Limited diffraction compensation
- Scalar Diffraction Propagation
 - Fraunhofer, Fresnel, Rayleigh-Sommerfeld
 - 2D complex wavefront
 - Propagation by summation of wavefronts
- Full-Wave Propagation
 - Finite Difference, Finite Element, Rigorous Coupled Wave, Boundary Integral
 - Computationally intensive



Increased Computation Speed ↑
↓ Increased Accuracy

Validity of Scalar Models



Examples: 50 μm Aperture, 200 μm Observation, $\lambda=850 \text{ nm}$, $\lambda=1550 \text{ nm}$

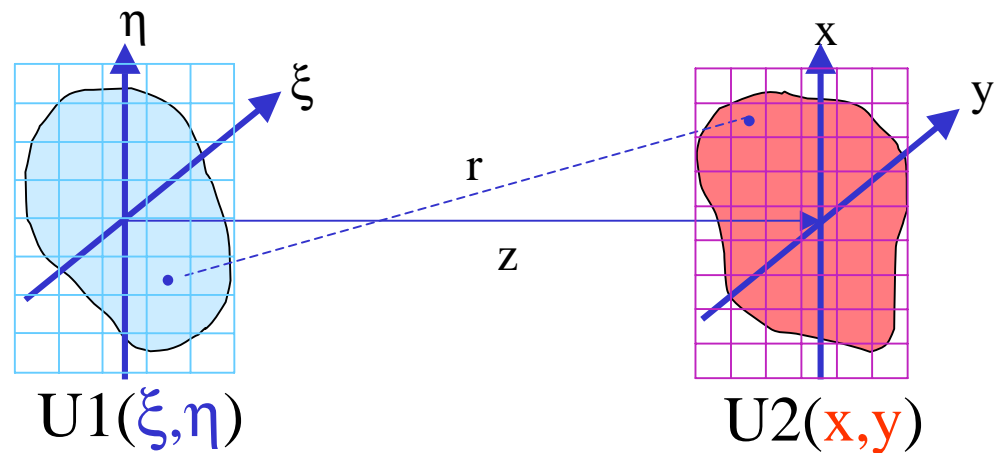
- Fraunhofer Approximation - Assume planar wavefronts
- Fresnel Approximation - Assume parabolic wavefronts
- Rayleigh-Sommerfeld Formulation - Spherical wavefronts

Rayleigh-Sommerfeld Formulation

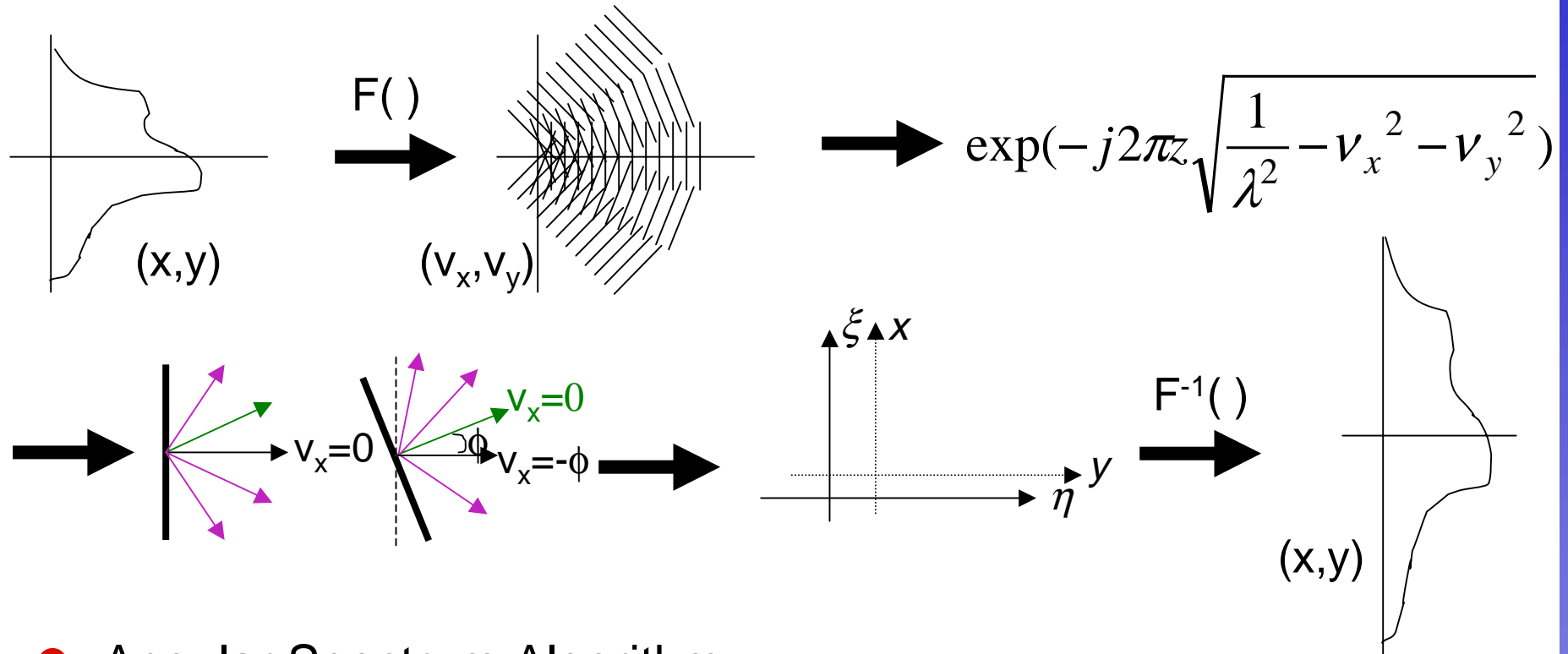
- Rayleigh-Sommerfeld Formulation
 - ⊙ Diffractive component $\gg \lambda$
 - ⊙ Distance to observation plane $\gg \lambda$

$$U_2(x, y) = \frac{z}{j\lambda} \iint U_1(\xi, \eta) \frac{e^{jkr}}{r^2} \partial\xi \partial\eta$$

- Implementation
 - ⊙ Huygens-Fresnel principle
 - ⊙ Direct Integration
 - Computational Order: $O(N^4)$



Rayleigh-Sommerfeld: Angular Spectrum



- Angular Spectrum Algorithm:

- Decompose wavefront into sum of angled plane waves using FFT
- Multiply with phase function for propagation
- Map spatial frequencies into tilted coordinate system
- Use Fourier shifting theorem for offset plane
- Sum plane waves into wave function with inverse FFT

Angular Spectrum and the Rayleigh-Sommerfeld Formulation

- Rayleigh-Sommerfeld Formulation:

$$U_2(x, y, z) = -\frac{1}{2\pi} \iint U_1(\xi, \eta, 0) \times \frac{\partial}{\partial z} \frac{e^{-jkr}}{r} \partial\xi \partial\eta$$

- Equivalently:

$$U_2(x, y, z) = \frac{1}{2\pi} \iint U_1(\xi, \eta, 0) \times \frac{e^{-jkr}}{r} \frac{z}{r} \left(jk + \frac{1}{r} \right) \partial\xi \partial\eta$$

$$r = \sqrt{z^2 + (\xi - x)^2 + (\eta - y)^2} \quad k = \frac{2\pi}{\lambda}$$

- Convolution: Complex wavefront & propagation through free-space:

$$U_2(x, y, z) = U_1(\xi, \eta, 0) \otimes \frac{1}{2\pi} \frac{\partial}{\partial z} \frac{e^{-jkr}}{r} \partial\xi \partial\eta$$

Angular Spectrum and the Rayleigh-Sommerfeld Formulation

- Convolution in the Spatial Domain is Multiplication in Frequency Domain:

$$U_2(x, y, z) = F^{-1} \{ F \{ U_2(x, y, z) \} \}$$

$$U_2(x, y, z) = F^{-1} \left\{ F \{ U_1(\xi, \eta, 0) \} \times F \left\{ \frac{1}{2\pi} \frac{\partial}{\partial z} \left[\frac{e^{-jkr}}{r} \right] \right\} \right\}$$

- The FFT of the complex wavefront is performed....

$$\begin{aligned} F \{ U(\xi, \eta, 0) \} &= \iint U_1(\xi, \eta, 0) \times \exp(-j2\pi(v_x \xi + v_y \eta)) \partial \xi \partial \eta \\ &= A_0(v_x, v_y, 0) \end{aligned}$$

- ...and is multiplied to the Free-Space transfer function

$$F \left\{ \frac{1}{2\pi} \frac{\partial}{\partial z} \left[\frac{e^{-jkr}}{r} \right] \right\} = \exp(-j2\pi z \sqrt{\frac{1}{\lambda^2} - v_x^2 - v_y^2})$$

Spatial Frequencies

- Spatial frequencies describe the propagating plane waves

$$A(v_x, v_y, 0) = \iint U(x, y, 0) \exp[-j2\pi(v_x x + v_y y)] \partial x \partial y$$

- Plane waves of the form (in 3D):

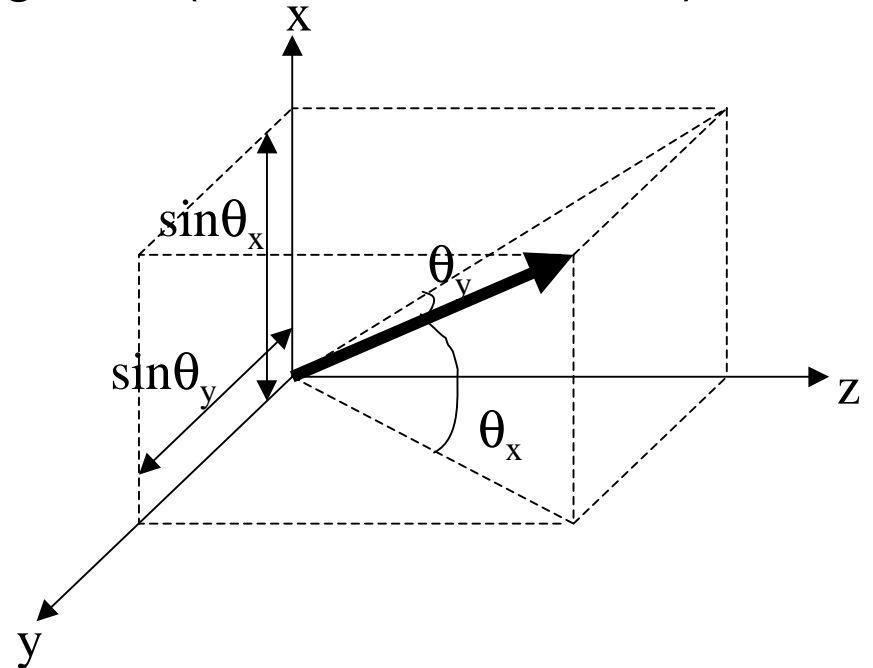
$$\exp[-j2\pi(v_x x + v_y y + v_z z)]$$

- Determine the direction of propagation (directional cosines)

$$v_y = \sin \theta_y / \lambda$$

$$v_x = \sin \theta_x / \lambda$$

$$v_z = \sqrt{\left(\frac{1}{\lambda}\right)^2 - v_x^2 - v_y^2}$$



Tilted Planes

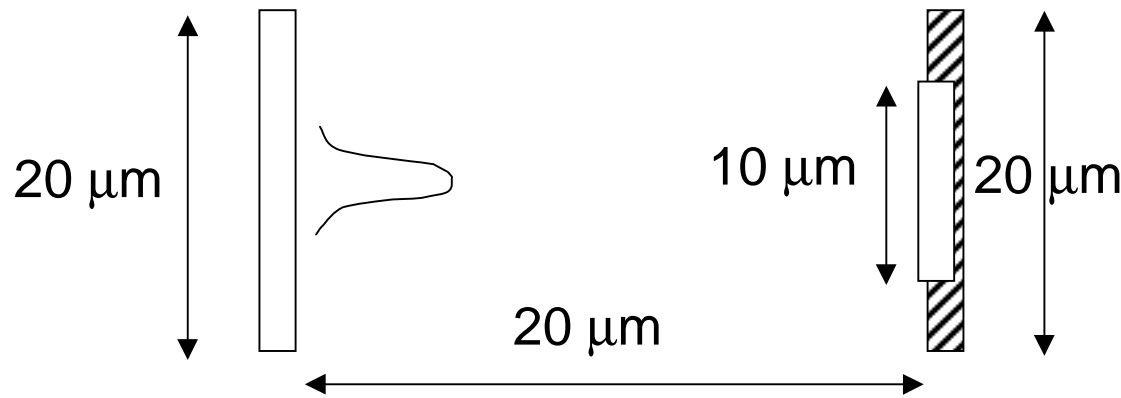
- When aperture and observation planes are not parallel:
 - Re-mapping of spatial frequencies
 - Valid since phase accumulation term does not change
- Re-mapping of spatial frequencies
 - Use well-known coordinate transfer matrices

$$(x, y, z)^T = R(\xi, \eta, z')^T \quad (v_x, v_y, v_z) = R(v_x', v_y', v_z')^T$$

$$R_{y\text{-axis}} = \begin{bmatrix} \cos \phi & 0 & -\sin \phi \\ 0 & 1 & 0 \\ \sin \phi & 0 & \cos \phi \end{bmatrix} \quad R_{x\text{-axis}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & \sin \theta \\ 0 & -\sin \theta & \cos \theta \end{bmatrix} \quad R_{z\text{-axis}} = \begin{bmatrix} \cos \rho & \sin \rho & 0 \\ -\sin \rho & \cos \rho & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



Computation Speed Comparison



System Parameters:

VCSEL (Gaussian Source):

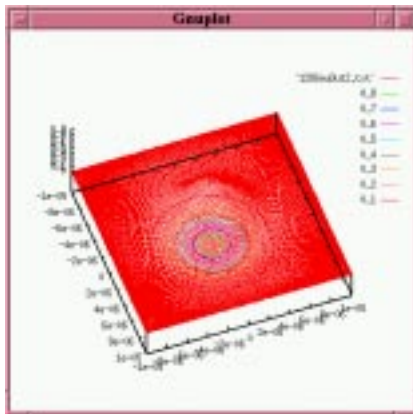
- 5 μm waist
- 1550 nm wavelength
- 20 μm bounding box

Distance:

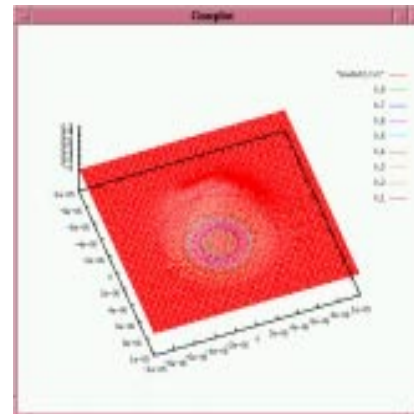
- 20 μm

Detector:

- 10 μm square detector
- 20 μm bounding box



Direct Integration(256x256)

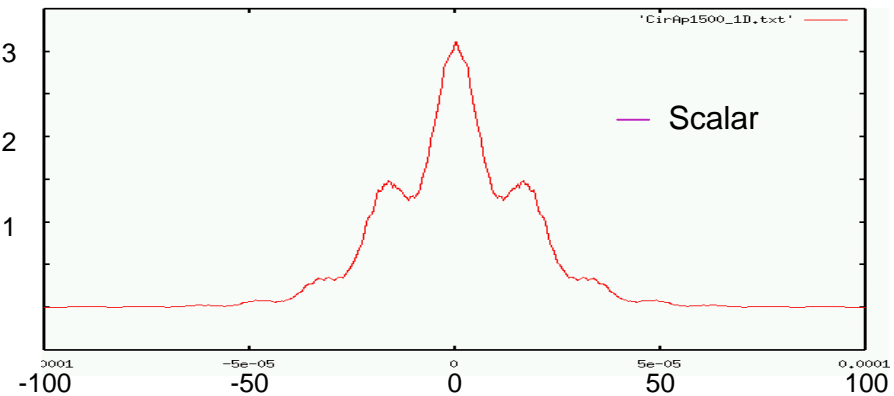
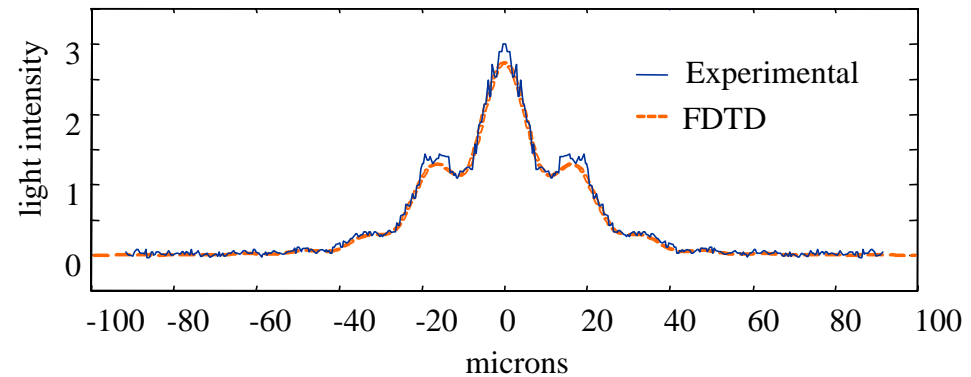
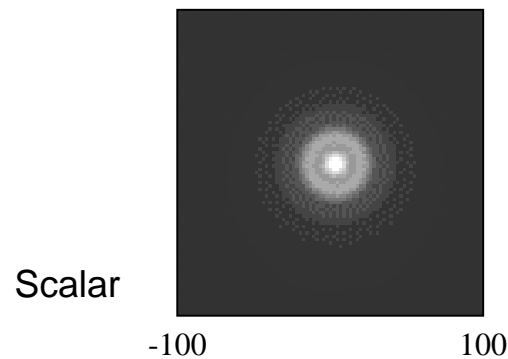
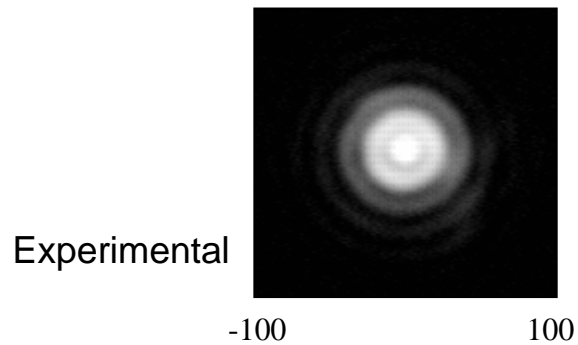


FFT(256x256)

	Fast Fourier Transform					Direct Integration (Gaussian Quadrature)				
	32	64	128	256	512	32	64	128	256	512
N (mesh side)	32	64	128	256	512	32	64	128	256	512
Computation (sec)	0.03551	0.1067	0.2744	1.8886	4.9675	1.8134	29.3992	455.841	7080	116480
% Power Error	0.13%	0.03%	0.01%	0.00%	0.00%	4.62%	0.97%	1.19%	0.14%	0.00%

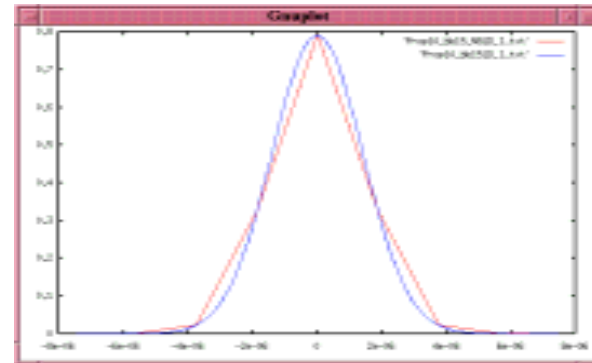
Experimental Validation

- Measure diffracted field 1.5mm past a 71 μm precision pinhole ($\lambda=632.8\text{nm}$)
 - Prather, University of Delaware
- Model using FDTD to validate electromagnetic code
 - Prather, University of Delaware
- Model using scalar theory

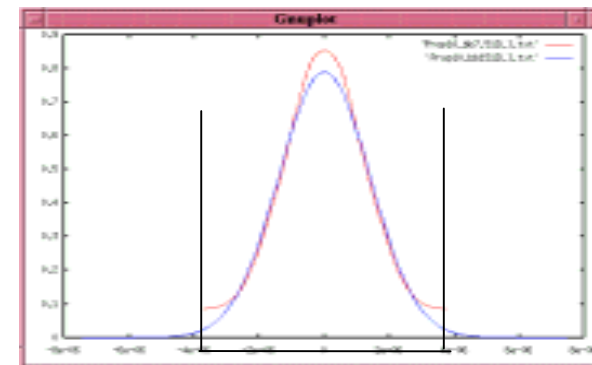


Error Analysis of Angular Spectrum

- Continuous Fourier transform theory
 - Angular spectrum method is exact Rayleigh-Sommerfeld solution
- Discrete Fourier transform theory
 - Continuous angular spectrum function is discretized
 - Errors are possible
- Common Errors:
 - Aliasing:
 - sampling too low to capture high frequencies
 - higher orders falsely placed in lower orders
 - Truncation:
 - signal cut-off on edges
- Errors Result in:
 - “Reflection” off of window
 - Undersampling



Aliasing



Truncation

Sampling of Angular Spectrum

- Fourier transform of x-y plane

- Plane waves: $\exp^{-j(v_x x + v_y y)}$
- Constant phase lines: $A = v_x x + v_y y$
- Angle: $\theta_k = \tan^{-1}\left(\frac{\lambda_x}{\lambda_y}\right) = \tan^{-1}\left(\frac{v_y}{v_x}\right)$
- Spatial wavelength

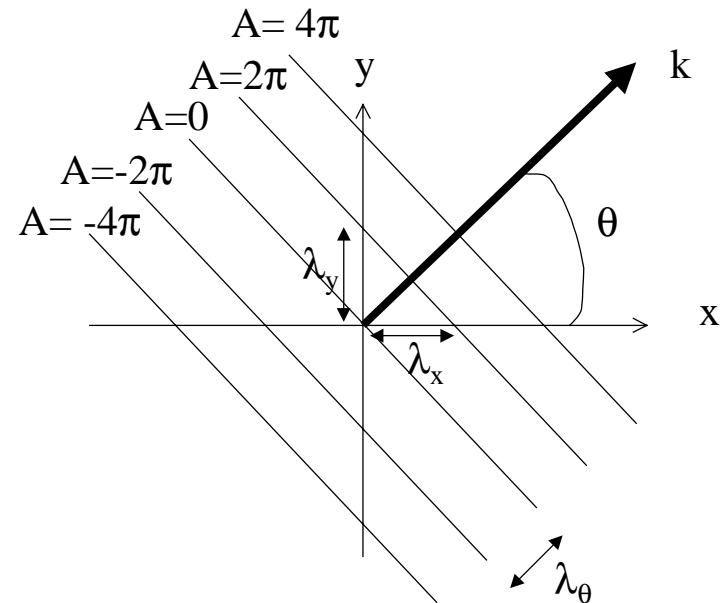
$$\lambda_\theta = \frac{1}{\sqrt{\frac{1}{\lambda_x^2} + \frac{1}{\lambda_y^2}}} = \frac{1}{\sqrt{v_x^2 + v_y^2}}$$

- Angular spatial frequency $k = \sqrt{v_x^2 + v_y^2}$

- For Plane waves propagating $\pm 90^\circ$ (half circle)

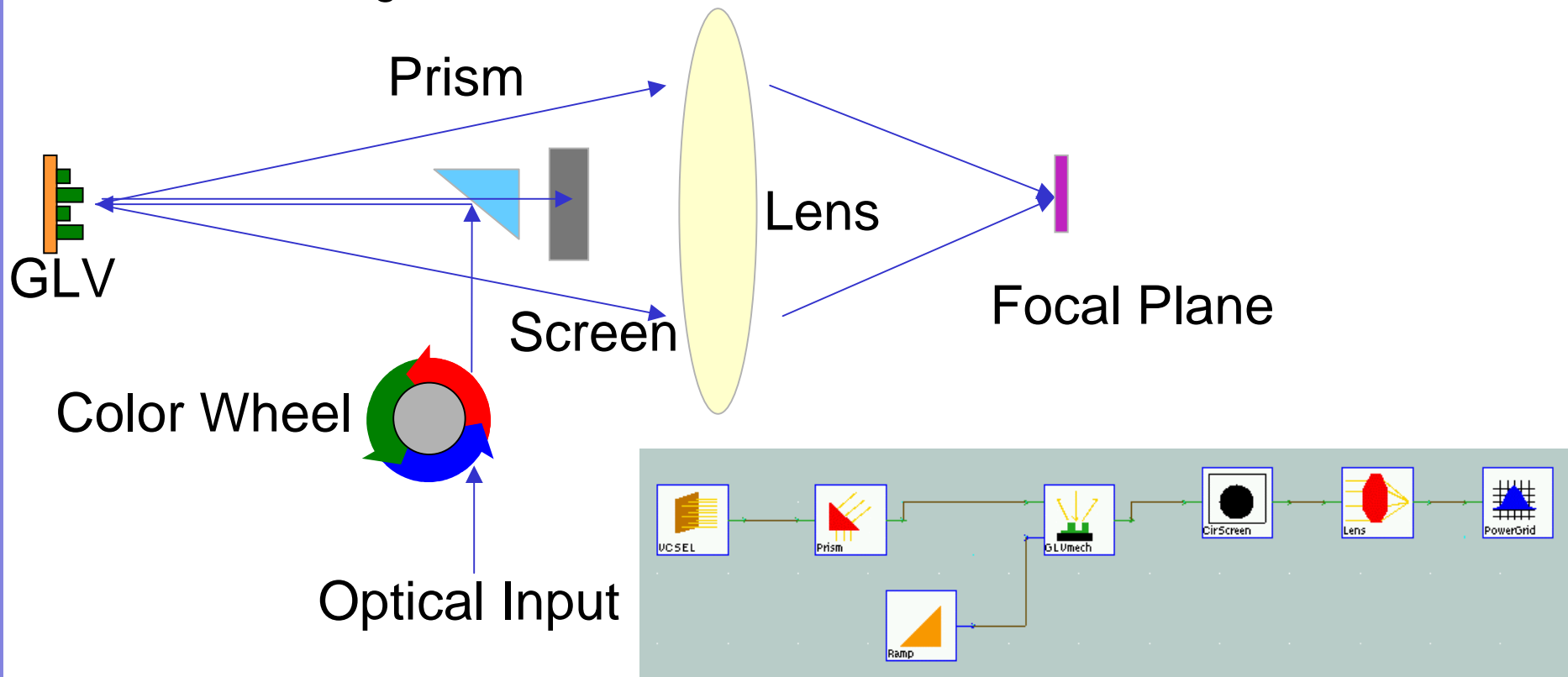
$$v_{x,y} = \frac{\sin \theta}{\lambda} = \frac{\sin \frac{\lambda}{2}}{\lambda} = \frac{1}{\lambda} \quad \lambda_\theta = \frac{1}{\sqrt{\frac{1}{\lambda^2} + \frac{1}{\lambda^2}}} = \frac{1}{\sqrt{2}} = \frac{\lambda}{\sqrt{2}}$$

- Using Sampling Theorem: $\Delta x, \Delta y \leq \frac{\lambda_{\min}}{2} = \frac{\lambda}{2\sqrt{2}}$

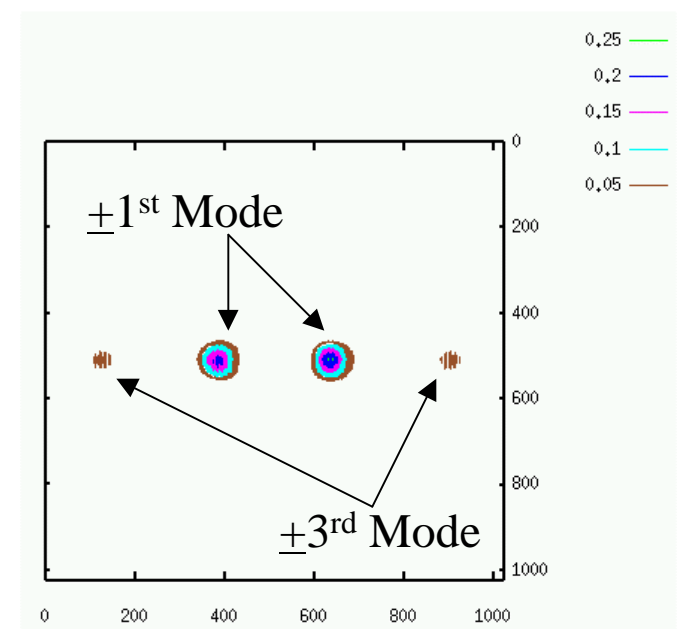
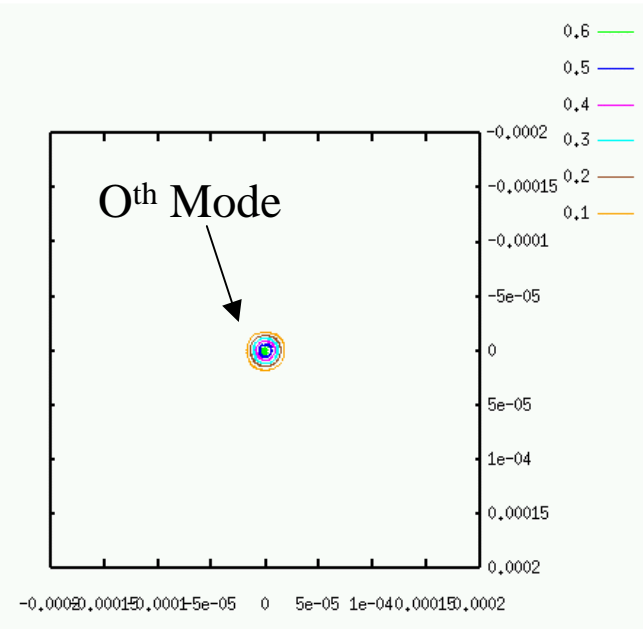
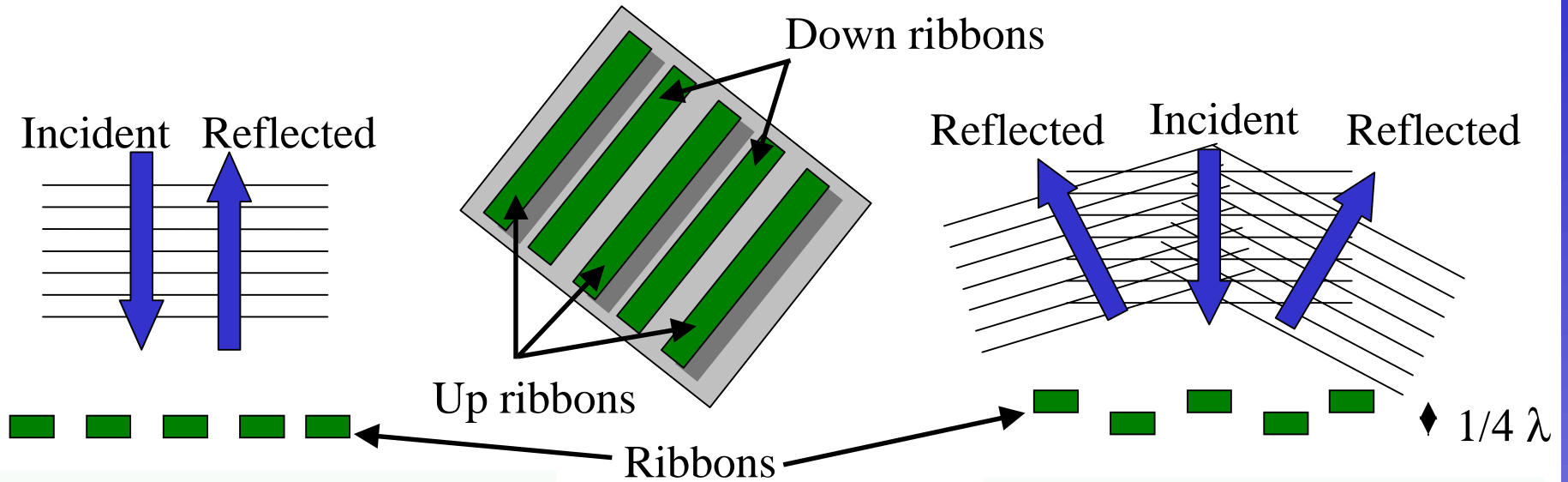


System-Level Simulation: Digital Projection System

- Digital Projection System using Grating Light Valve
 - If pixel is “on”, light is reflected off GVL at an angle, propagating through the lens to the focal plane
 - If pixel is “off”, light is reflected straight off GVL into the absorbing screen

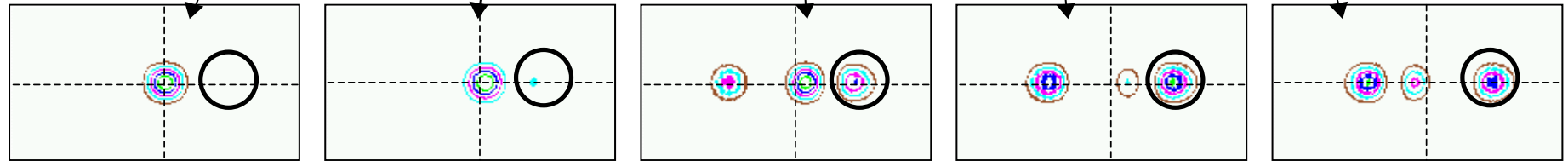
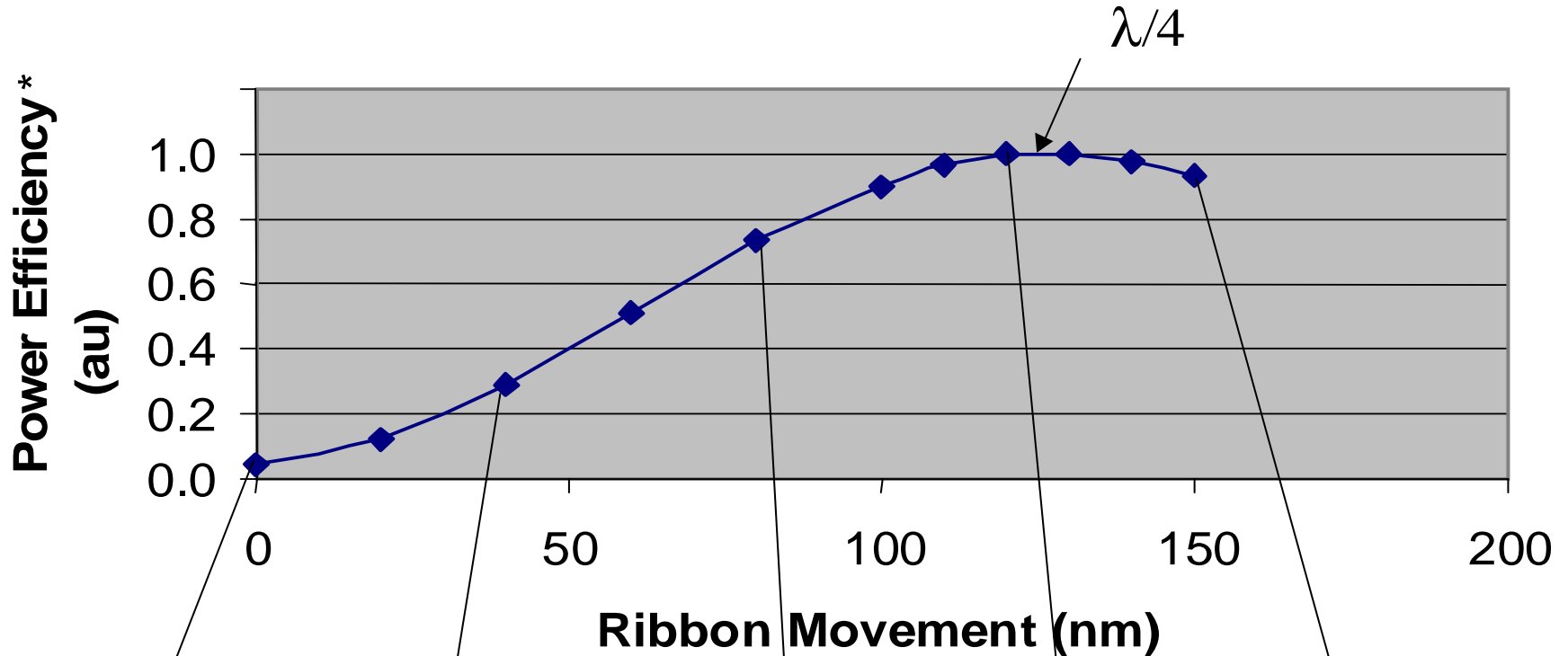


Grating Light Valve (GLV)



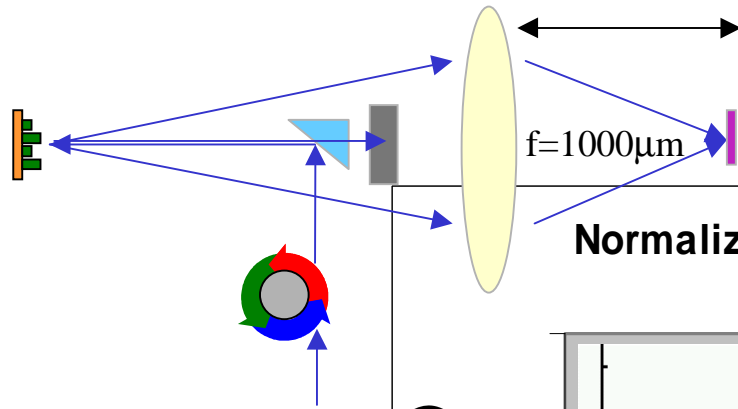
Ribbon Movement vs. Power Efficiency

Ribbon Movement vs. 1st Mode Power Efficiency

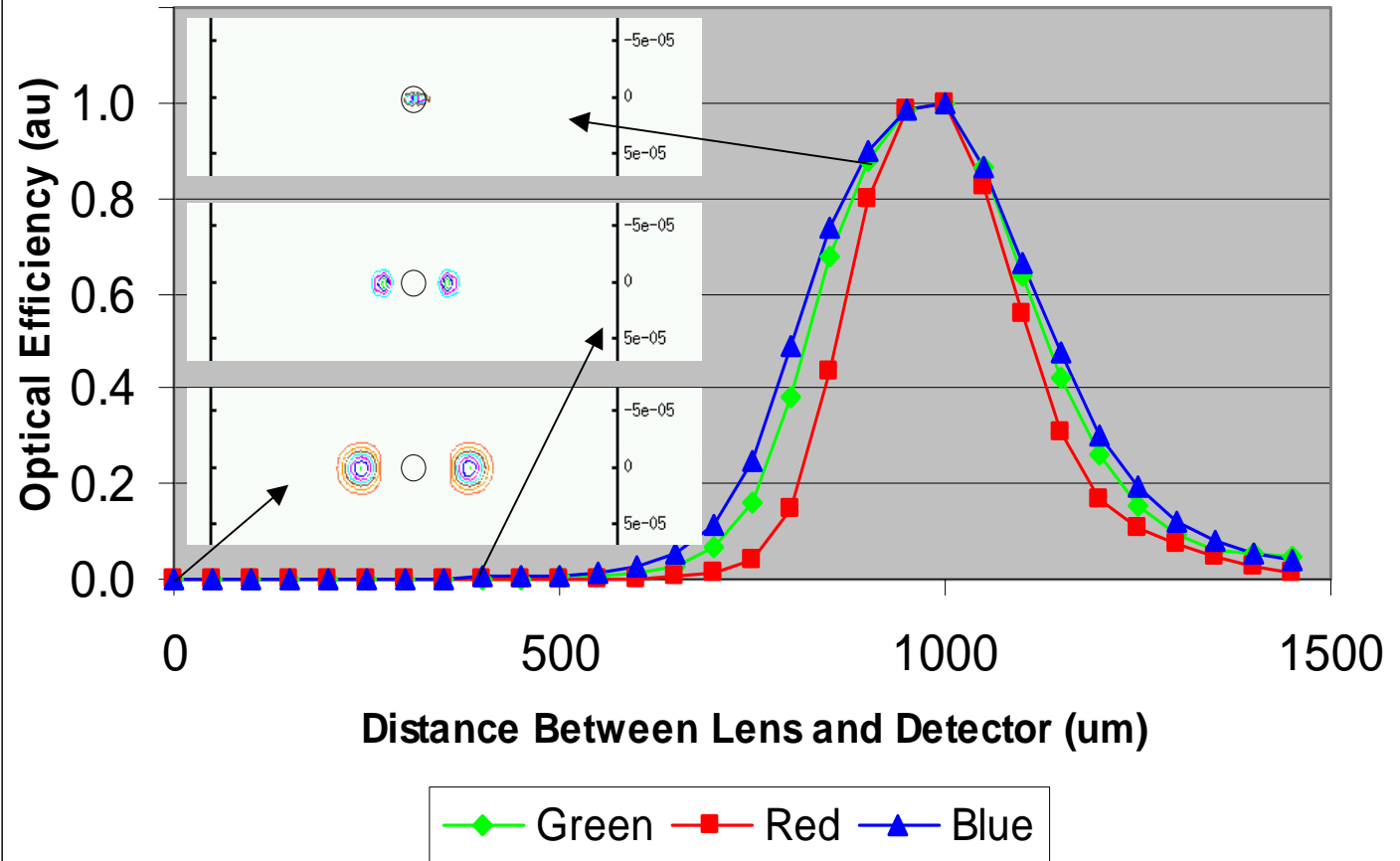


*Encircled energy (12.5 μm radius) in the +1 mode

Multi-Wavelength Simulation



Normalized Power Efficiency vs. Distance Between Lens and Detector Plane



Summary and Future Work

- Optical micro-systems and system-level CAD
- Rayleigh-Sommerfeld Formulation required for accurate optical propagation modeling
- Angular spectrum method
 - ⊙ Reduce $O(N^4)$ Direct Integration to $O(N^2 \log N)$ FFT
 - ⊙ Convolution of the complex wavefront and free-space transfer function
 - ⊙ Achieve the speed of the Fraunhofer approximation, with the accuracy of the Rayleigh-Sommerfeld formulation
- System-level simulations and validation
- Future work
 - ⊙ Further error prediction
 - ⊙ Determine limitations of Rayleigh-Sommerfeld scalar technique for optical micro-systems