Nonlinear Model Order Reduction Using Remainder Functions

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1. Introduction

This paper describes a novel approach to the problem of model order reduction (MOR) of very large nonlinear systems. We consider the behavior of a dynamic nonlinear system as having two fundamental characteristics: a global behavioral "envelope" that describes major transformations to the state of the system under external stimuli and a local behavior that describes small perturbation responses. The nonlinear low order envelope function is generated by using the remainders from the coalescence of projection bases taken through a space-state sample. A behavioral model can then be expressed as the superposition of these two descriptions, operating according to the input stimuli and the current state value.

The global behavior describes major transformations to the state of the system under external stimuli and the local behavior describes small perturbation responses. Local effects are captured by regions through a set of linear projections to a reduced state-space while global effects are captured by examining the non-commonalty among these projections. These "remainders" are used to build a modulation function that will generate the required dynamic changes in the common linear projection.

The advantage of the envelope representation for strongly nonlinear systems is that it simplifies the complexity of the model into a two-part problem. Depending on the complexity or cost of the behavioral separation procedure, it can be repeated recursively.

2. New model extraction definition

In contrast to current approaches for the generation of nonlinear compact models based on an assembly of piecewise linear models [1] (as shown on the left side of figure 1) we extract a nonlinear function that modulates a projection base, to account for its variation throughout the region of interest in the state-space (shown on the right side of figure 1). In the figure, we are representing the different instances of the subspaces where the desired linear functions are defined, by corresponding linear projections V_i (shown as *2D* vectors for simplicity).

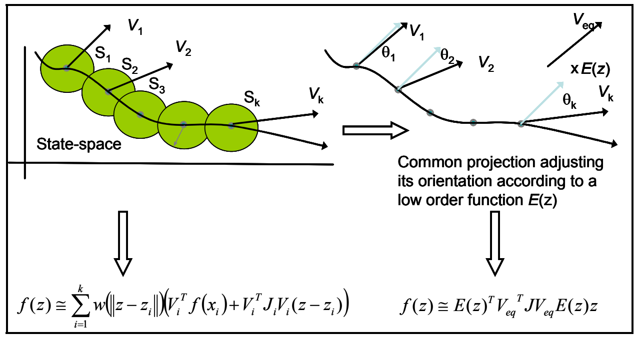

Figure 1, Envelope kernel, *E(z)*, modifying a linear model description of the nonlinear function *f(z)*

In our model, we consider that the variations in the tangent hyperplane orientations can be described by a set of closed confining functions that depend on only a few parameters. The resulting model can then be abstracted as those functions modulating the behavior of a shared linear base corresponding to a common linear subspace. The linear subspace base is the one that maximizes the commonality of all the projection bases that are sampled. Furthermore, the deviations from this common sub-space or "remainders" are the basis for the generation of the desired modulating, or nonlinear envelope function. This procedure for MOR can be described in the following three steps:

2.1. Region decomposition/projection extraction

In the first step, we divide the state-space into neighboring sub-regions (shown in figure 2 as S_i). The proximity between these regions can be defined using the geometrical concept of Euclidian distance.

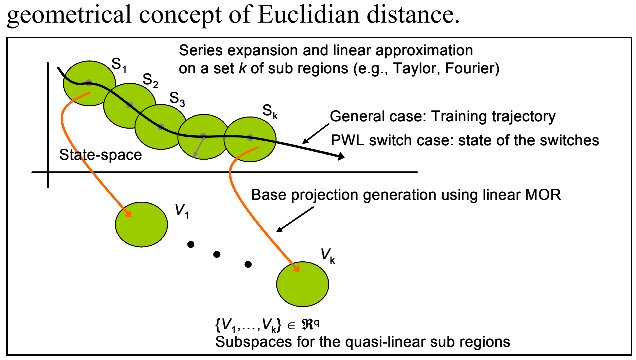

Figure 2, Region decomposition of the state-space

We then approximate the solution on these sub-regions considering their quasi-linear behaviors. This allows us to use one of the well known linear projection extraction algorithms to find a suitable projection base V_i for each sub-region S_i [1][2]. We determine an optimal radius for each quasi-linear region using the Hessian as a metric to judge the linearity of the region. This technique shows a promising method for the optimal generation of this set of samples.

2.2. Coalescence of quasi-linear region bases

Next, our projection set $\{V_1,\ldots,V_k\} \in \Re^q$ is used to extract a projection base, V_{eq}, that maximizes the correlation among elements in this set. Geometrically, we are finding the projection base that contains the maximum of all the other projections in the set, shown in figure 3. We then use this common base to decompose the set and obtain the remainders, which are orthogonal to V_{eq} and represent the differences between sub-regions. This result is given by: $V = [V_{eq} \ \ \text{span}\left(V_{1r}^{\perp},..,V_{kr}^{\perp}\right)]$

where $V_{eq} \in \Re^{n\mathrm{x}r}$ corresponds to the common linear projection base for the reduced state-space of size r, and $\{V_{ir}^{\perp} \in \Re^{n\mathrm{x}(q-r)}\text{: } 1<i<k\}$ corresponds to the set of k orthogonal remainders.

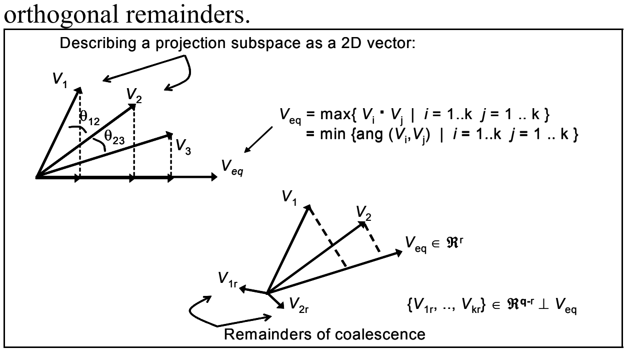

Figure 3, Projection base coalescence

It is important to note that the model order reduction for each of the quasi-linear regions has been initially targeted to a size $q<<n$, however, the effective linear reduction achieved corresponds to the size r and the difference on the order $(q-r)$ is absorbed by the nonlinear remainders.

While the idea of merging the set of projection bases to obtain an expanded base that contains the resulting solutions in a "suitable form" has been proposed by several authors [1],[2] the importance of using the differences in the set, captured through the proposed remainders, to improve the model final accuracy has not been explored.

2.3. Nonlinear envelope extraction

Finally, we generate the nonlinear function that captures the combined behavior of the remainders. This envelope function can be obtained using well know control identification techniques such as regression analysis [3].

In our case, the values of the projection remainders above are used to fit a template (regressor function) for the nonlinear envelope.

The outcome from this can be represented in two different forms as shown in figure 4: either as a dispersed variation, ΔV_i, which would be used for the generation of the envelope function to modulate V_{eq}.; or, as a localized variation, V_{dif_i}, in an identifiable space size $(q-r)$, which uncovers a hidden linear/nonlinear block structure in the original representation. This also can be used as an identification technique to trigger further decomposition of the original system in a hierarchical structure.

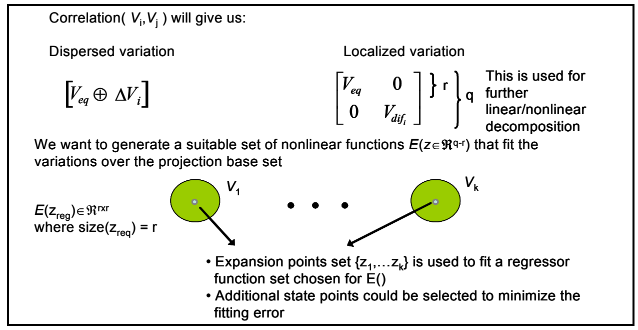

Figure 4, Envelope kernel generation

We will present a comparison of this technique against standard PWL methods [1][2]. Initial results show a lower dimensionality for comparable accuracy. The generation cost however can be higher than similar techniques. Undergoing research is aimed to improve these metrics.

3. Discussion

In hierarchical systems, for every new component, bottom up model building must precede top down design. For new technologies, where models do not exist, methodologies such as the one presented here will facilitate the extraction of compact models from low level abstractions of new devices into system level representations that are efficient and inexpensive to evaluate.

Acknowledgment

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