# An Efficient Optical Propagation Technique for Optical MEM Simulation

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## ABSTRACT

As designers become more aggressive in introducing optical components into micro-systems, additional capabilities are required for modeling and simulation tools. Common optical modeling techniques are not applicable for most optical micro-systems, and techniques that are valid are computationally slow. In this paper, we introduce an angular spectrum propagation modeling technique that greatly reduces computation time while maintaining the accuracy of the full scalar formulation. We present simulations of light propagating through optical MEM components and show the advantages of this optical propagation method and the integration of the technique into a system-level multi-domain CAD tool.

***Keywords:*** optical MEMS, MOEMS, angular spectrum, optical propagation, CAD

## 1 INTRODUCTION

It is well known that optics can provide advantages to micro-systems in terms of speed, bandwidth and reduced power [7]. However, by adding the optical domain to micro-systems, many new challenges are introduced in system design. In these multi-domain systems, optical effects, such as diffraction, interference, and scattering, are critical to the success or failure of the designs. Therefore, modeling and simulation are crucial early in the design stage.

Currently, multi-domain micro-systems are simulated by domain-specific tools, using component level models that are performed at the physical level. In contrast, system-level tools are designed to include multiple domains and allow efficient system simulation, modeling components by their functionality rather than their physical construction. Established MEM system-level modeling tools exist, as physical device models are extracted to the system-level. However, optical propagation models are not easily integrated into these tools. In this paper, we describe an optical propagation technique suitable for system-level multi-domain micro-system CAD tools.

When light interacts with the small feature sizes of micro-systems, many common optical propagation modeling techniques are invalid, and full vector or scalar solutions to Maxwell's equations are required [3]. However, these optical modeling techniques are computationally and memory intensive, leaving interactive design, between system designer and CAD tool, almost impossible. As more optical components are introduced into micro-systems, the need for accurate and efficient simulation tools increase. Therefore, the difficulty in determining an optical propagation technique suitable for system-level CAD tools for modeling optical MEM systems is that the optical model must be rigorous enough to support micro-system dimensions and also be computationally efficient.

In this paper, we introduce an optical propagation model that provides valid results for micro-systems, with a computational algorithm that allows for interactive CAD design. We first present a brief background of optical propagation modeling, from which we find valid techniques for optical MEM system modeling. Next, we provide a description of the angular spectrum technique used to greatly reduce the computational load of optical modeling. We follow with a presentation of example optical MEM systems, with simulations and analyses. We conclude with a summary and future work.

## 2 SCALAR OPTICAL PROPAGATION

Optical propagation can be modeled completely by the solution of Maxwell's equations for both the electric field vector, $\vec{E}$, and the magnetic field vector, $\vec{H}$ [2]. The computation is performed in 3D, as each vector is composed of *x*, *y*, and *z* values. This method is valid for optical modeling in micro-systems, however, the computation time and memory requirements are extremely demanding.

To reduce the computational resource requirements, a scalar representation is commonly used. Scalar optics are defined by summarizing vectors $\vec{E}$ and $\vec{H}$ into a single complex scalar, $U$. No longer are we solving in 3D, as the scalar function represents a complex 2D wave function. This replacement is valid if the propagation occurs in a dielectric medium. Further, the propagation medium needs to be linear, isotropic, homogenous, nondispersive, and nonmagnetic. Propagation through free-space meets these requirements.

Similar to the vector solution, this complex scalar must also satisfy the wave equation, known as the Helmholtz equation: $(\nabla^2 + k^2)U = 0$, where, the wave number, $k = (2\pi)/\lambda$. With use of Green's theorem, the Rayleigh-Sommerfeld formulation can be derived from the wave equation for the propagation of light in free-space [2]:

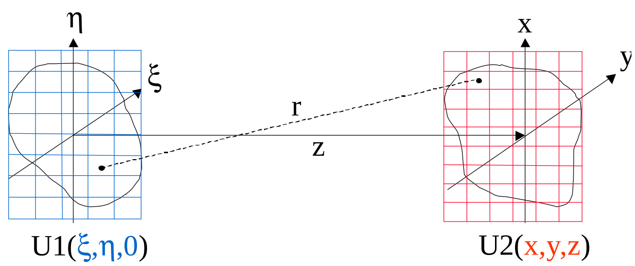

Figure 1: Aperture and Observation Coordinate System

$$U(x, y, z) = \frac{z}{j\lambda}\iint_{\Sigma} U(\xi, \eta, 0)\frac{\exp(jkr)}{r^2}\partial\xi\partial\eta$$

where, $r = \sqrt{z^2 + (x-\xi)^2 + (y-\eta)^2}$, $\Sigma$ is the area of the aperture, and $z$ is the distance that the light is propagated from an aperture plane ($z$ = 0) to an observation plane. It is assumed that the two planes are parallel, with coordinate systems (ξ,η,0) in the aperture plane and ($x$,$y$,$z$) in the observation plane, as seen in Figure 1. The formulation is valid as long as both the propagation distance and the aperture size are greater than the wavelength of light. These restrictions are based on the boundary conditions of the Rayleigh-Sommerfeld formulation, and the fact that the electric and magnetic fields cannot be treated independently at the boundaries of the aperture [2].

To compute the complex wave front at the observation plane, we divide both the aperture plane and the observation plane into gridded meshes. The planes are commonly meshed into NxN regions, where N is the number of mesh points along the side of a square. Using a direct integration technique we have successfully shown results using the Rayleigh-Sommerfeld method [3].

However, this direct integration method is computationally intensive. For each point on the observation plane, the sum of the entire aperture plane, with respect to the observation point, is required. In other words, for each of the $N^2$ points on the observation plane, all $N^2$ points on the aperture plane are summed. Therefore, the computational order of this direct integration algorithm is $O(N^4)$.

The Fraunhofer and Fresnel, or far and near field, approximations are made from the Rayleigh-Sommerfeld formulation. With the far field assumptions, the wavefront of the observation plane can be calculated by the Fourier transform of the aperture plane [2]. This greatly reduces the computational resources needed to solve the direct integration of the Rayleigh-Sommerfeld formulation. However, in previous research [3], we have shown that these techniques are not generally valid for optical micro-systems. Therefore, only the full scalar equations, without approximations, will provide the validity and accuracy that is required for optical propagation in micro-systems.

In the interest of reducing this computational load, we look to recast the full Rayleigh-Sommerfeld formulation using the angular spectrum technique, in order to take advantage of the Fourier transform.

## 3 ANGULAR SPECTRUM OF LIGHT

As an alternative to direct integration over the surface of the wavefront, the Rayleigh-Sommerfeld formulation can be solved using a technique that is similar to solving linear, space invariant systems with the use of a Fourier transform. In this optical case, we use an angular transform to identify the components of the angular spectrum, which are plane waves traveling in different directions away from the surface [2]. By using the Fourier transform we reduce the complex optical wavefront into a set of simple exponential functions, plane waves. This technique is valid for centered parallel planes separated by a distance $z$, therefore, the aperture and observation coordinates are the same: ξ =x and η=y. In this discussion, we use the coordinates (x,y,z).

The complex wave function U(x,y,0) has a 2D Fourier transform in terms of angular frequencies, $v_x$ and $v_y$.

$$F\{U(x, y, 0)\} = A(v_x, v_y, 0)$$

$$A(v_x, v_y, 0) = \iint U(x, y, 0)\exp[-j2\pi(v_x x + v_y y)]\partial x\partial y$$

where, $v_x = \frac{\sin\theta_x}{\lambda}$ $\qquad v_y = \frac{\sin\theta_y}{\lambda}$.

$\sin(\theta_x)$ and $\sin(\theta_y)$ are the directional cosines of the plane wave propagating from the origin of the coordinate system.

The inverse Fourier transform is:

$$U(x, y, 0) = \iint A(v_x, v_y, 0)\exp[j2\pi(v_x x + v_y y)]\partial v_x\partial v_y$$

In the above equations, $A$ is the complex amplitude of the plane wave decomposition defined by the specific angular frequencies.

To propagate the complex wave function, U(x,y,0) to a parallel plane, U(x,y,z), the complex amplitude is multiplied by a phase term, β. β is computed by satisfying the Helmhotz equation [2]. This results in:

$$A(v_x, v_y, z) = A(v_x, v_y, 0)\beta \qquad \beta = \exp\left(jz2\pi\sqrt{\frac{1}{\lambda^2} - v_x^2 - v_y^2}\right)$$

This phase term describes the distance that each of the plane waves travels due to the propagation between the parallel plates. Therefore, the wave function after propagation can be solved with the following inverse Fourier transform:

$$U(x, y, 0) = \iint A(v_x, v_y, 0)\beta\exp[j2\pi(v_x x + v_y y)]\partial v_x\partial v_y$$

It is interesting to note that the above equation is simply the convolution of two functions. The first function is the input complex wave function, and the second is the propagation effect.

Removing the restrictions of only propagating between parallel planes sharing a common center has been the goal of recent research. Tommasi and Bianco have determined how to propagate to a plane that is tilted with respect to initial plane [5]. Delen and Hooker have determined a way to allow offsets in the observation plane [1]. We summarize these two methods next.

For arbitrary angles between a normal plane, U(ξ,η,ζ), and a tilted plane, U(x,y,z), a remapping of the normal plane's spatial frequencies into the tilted plane's spatial frequencies is required. This mapping is possible since the phase accumulation term does not change when the waves propagate to an observation plane not normal to the aperture plane. It can be found that the rotational matrix, M, used to relate coordinate positions (ξ,η,ζ) to (x,y,z), can also be used to relate spatial frequencies [5]:

$$(x, y, z)^t = M(\xi, \eta, \zeta)^t \qquad (v_\xi, v_\eta, v_\zeta)^t = M(v_x, v_y, v_z)^t$$

For an observation plane whose center is offset from the propagation axis of the aperture plane, the Fourier shifting theorem can be used to solve for the complex wave function [1]. The coordinate systems of the aperture, (ξ,η,ζ), and observation plane, (x,y,z), need to be related by:

$$x = \xi - x_0 \qquad y = \eta - y_0$$

With this relation between the offset of the coordinate systems, the function for free-space propagation between offset planes is:

$$U(x, y, 0) = \iint A'(v_x, v_y, z)\beta \exp[j2\pi(v_x x + v_y y)]\partial v_x \partial v_y$$

where, $A'(v_x, v_y, z) = A(v_x, v_y, 0)\exp[j2\pi(v_x x_0 - v_y y_0)]$

In summary, the angular spectrum technique for modeling propagation between the aperture and observation plane is implemented by the following. First, the forward Fourier transform is applied to the aperture surface. This is then multiplied by the propagation phase term. If tilts are present, the remapping of spatial frequencies occurs. If offsets between the planes occur, then the shifting theorem is applied. Finally, the inverse Fourier transform is applied, and the complex wavefront on the surface of the observation plane is obtained.

The advantage of using the angular spectrum to model light propagation is that the method is based on Fourier transforms. In CAD tools, the Fourier transform can be implemented by one of the numerous Fast Fourier Transform (FFT) techniques [4]. The computational order of a 2D FFT is $O(N^2 \log_2 N)$, much faster than $O(N^4)$ for the direct integration method. We show this speed increase later through example.

Like the direct integration technique, the FFT technique requires the aperture and observation planes to be discretized into NxN meshes, where N is the number of mesh points on the side of the plane. Equal spacing meshing is required, and for ease of the FFT algorithm, a power of 2 is suggested for the number of mesh points. In this discussion, we assume that the aperture and observation planes are meshed with the same N, however, this is not a requirement.

Choosing the size and resolution of the mesh is critical for accuracy and validity of the angular spectrum method. For accurate results, the observation plane needs to have sufficient zero padding around the optical waveform, since the edges of the computation window act as reflectors. If significant optical power reflects off the wall, interference between the propagating beam and these reflections can occur, resulting in inaccurate simulations. The resolution of the aperture and observation plane meshing should be at least λ/2 to ensure plane waves propagating from aperture to observation plane in a complete half circle, that is, between 90 and -90 degrees [5].

## 4 SIMULATIONS

To examine the speed-up of using the angular spectrum method, compared to direct integration, we simulate a Gaussian beam propagating in free-space using both methods. In these simulations, a 5 μm (diameter) Gaussian shaped beam with a wavelength of 1550 nm propagates 20 μm to a 10 μm square detector, as seen in Figure 2. Note that at 20 μm this system is in the "near-near" field, requiring the calculation of the complete Rayleigh-Sommerfeld formulation for accurate modeling. Simulation results, in terms of total computation time and percent difference of power detected on the detector compared with the N=512 "base case", are reported in the following table.

| N (mesh side) | 32 | 64 | 128 | 256 | 512 |
|---|---|---|---|---|---|
| | Angular Spectrum (FFT) | | | | |
| Computation (sec) | 0.03551 | 0.1067 | 0.2744 | 1.8886 | 4.9675 |
| % Power Error | 0.13% | 0.03% | 0.01% | 0.00% | 0.00% |
| | Direct Integration (Gaussian Quadrature) | | | | |
| Computation (sec) | 1.8134 | 29.3992 | 455.841 | 7080 | 116480 |
| % Power Error | 4.62% | 0.97% | 1.19% | 0.14% | 0.00% |

* Run on dual 1GHz processors running Linux with 2GB of RAM

To show system level simulations, we perform a transient simulation of an absorbing screen moving in front of a propagating Gaussian beam. This results in the beam being clipped by various amounts. This moving screen example is similar to a mirror moving in an optical MEM switch. As the mirror switches optical power from one state to the other, the intermediate optical state and switching time can effect the success of the switch. In this example, the same 5 μm Gaussian source is used as in the previous example and N=256. The light propagates to an observation plane 50 μm from the source. A system diagram, viewed from the top, with dimensions are shown in Figure 3. Again, with these small system dimensions, the full Rayleigh-Sommerfeld formulation is required for accurate simulation.

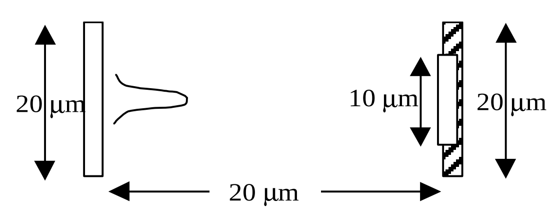


Figure 2: Gaussian Propagation Example System

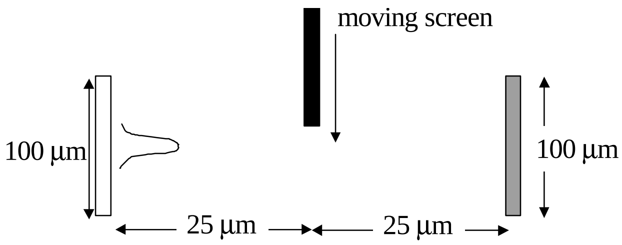


Figure 3: Moving Screen Example System

Simulation results of the light striking the observation plane for four clipping cases in the transient simulation are presented in Figure 4. The figure includes 20x20 µm intensity contours on the observation plane and also a diagram of the screen position in relation to the Gaussian beam. Obviously, power is lost when the beam is clipped. However, it is interesting to note the shape of the Gaussian beam as the beam is clipped. As the beam is slightly clipped, the propagating beam still appears to be Gaussian. At small clipping cases, the beam deforms, as the beam diffracts around the screen. In larger clipped cases, the direction of the Gaussian beam appears to change as the center of the beam moves off-axis.

## 5 SUMMARY AND FUTURE WORK

In this paper, we have demonstrated a promising optical propagation technique that can greatly speed-up simulation of optical micro-systems. We have shown how to achieve full scalar diffraction with the use of a FFT, by using an angular spectrum. This reduces an $O(N^4)$ direct integration problem to $O(N^2 \log_2 N)$. With the angular spectrum method used for the full Rayleigh-Sommerfeld formulation, we can achieve the computational speed of the far field approximation, without the need of any approximations. This technique has been implemented into our system-level CAD tool, allowing us to achieve interactive CAD development and simulation of complex optical MEM systems.

However, even with an efficient optical propagation model in our system-level CAD tool, microsystems with large arrays of optical beams is difficult to simulate, since the entire optical wavefront surface is required to be meshed. This leads to a large value of N and large memory requirements. Therefore, high-level optical propagation methods, such as 1D Gaussian beam propagation, are advantageous for system-level modeling. However, Gaussian models can not support the diffraction and clipping common in microsystems. As presented in the last example, we can characterize diffracted and clipped Gaussian beams with our fast optical propagation model. Our plans are to use these kind of results and extract the complex optical wavefront into a higher-level optical model composed of circular and elliptical Gaussian beams, or a more complex representation with multiple Gaussian beams, and use this hybrid technique for large system simulations.

## 6 ACKNOWLEDGMENTS

We would like to acknowledge the support of NSF grant CCR-9988319 and DARPA grant AFOSR F49620-01-1-0536 under the NeoCAD program.

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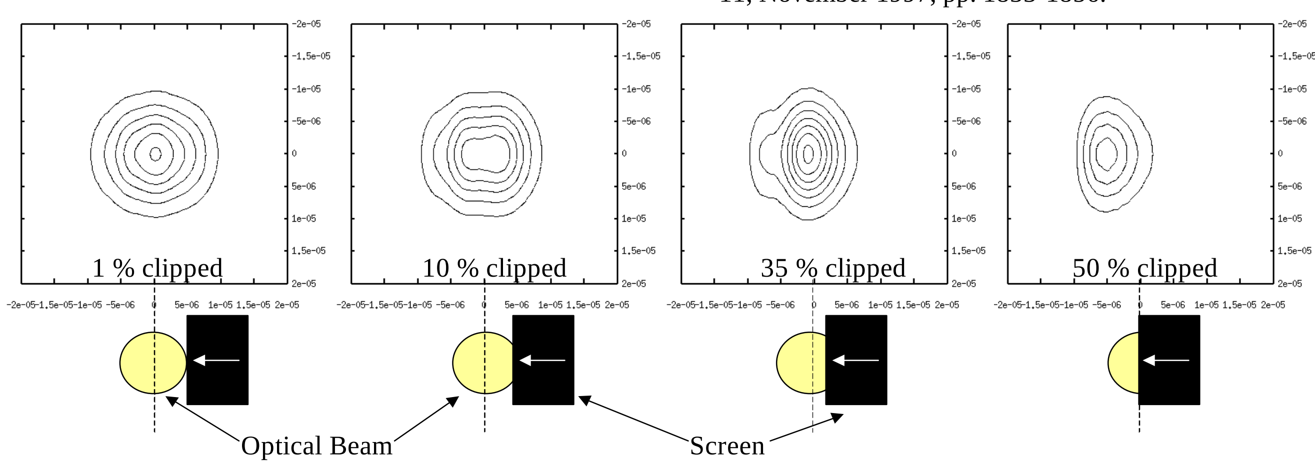


Figure 4: Intensity Contours of Clipped Gaussian Beam